Sol^n 1. Let us define r.v. $X_i$ corresponding to the ith person as follows:

$$X_i = \begin{cases} 1 & \text{if the person selects his/her own coat} \\ 0 & \text{o/w} \end{cases} \rightarrow 1 \text{ mark.}$$

(indicator r.v.)

(distribution of indicator r.v.) $P(X_i = 1) = \frac{1}{n}$ (each assignment is equally likely)

$P(X_i = 0) = 1 - \frac{1}{n}$ — 1 mark.

We want to find $E(X)$ where (identifying X in terms of indicators)

$X = X_1 + X_2 + \cdots + X_n$ — 1 mark.

so that X is the number of people who got their coats back.

By linearity, we have

$$E(X) = E(X_1 + \cdots + X_n) = \sum_{i=1}^{n} E(X_i).$$ — 1 mark (using linearity)

Also, $E(X_i) = 1 \times \frac{1}{n} + 0 \times \left(1 - \frac{1}{n}\right) = \frac{1}{n}$

or you can identify $X_i \sim Ber\left(\frac{1}{n}\right)$

$\Rightarrow E(X_i) = \frac{1}{n}$.

$\therefore E(X) = n \cdot \frac{1}{n} = 1.$ — 1 mark (answer).

Solⁿ 2. False — 2 marks. (claiming False)

— give a counter example to disprove:

$P(X+Y = 12, X-Y = 1) = 0$ $X+Y = 12 \Rightarrow X-Y = 0.$

$P(X+Y = 12) = \frac{1}{36}$ → Favourable outcome: $(6,6)$.

$P(X-Y = 1) = \frac{5}{36}$ → Favourable outcomes: $(2,1), (3,2), (4,3), (5,4), (6,5)$.

3 marks (explanation)

$\therefore P(X+Y = 12, X-Y = 1) \neq P(X+Y = 12) \cdot P(X-Y = 1)$

$\Rightarrow X+Y$ and $X-Y$ are not independent.

Knowing the total is 12 tells us the difference is 0, so r.v.s provide information about each other.